I.S.S.N: 1885-6888

ECONOMIC ANALYSIS WORKING PAPER SERIES

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Working Paper 1/2012

UNIVERSIDAD AUTONOMA DE MADRID

DEPARTAMENTO DE ANÁLISIS ECONÓMICO: TEORÍA ECONÓMICA E HISTORIA ECONÓMICA

A POOLING APPROACH TO JUDGMENT AGGREGATION

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Abstract

The literature has focused on a particular way of aggregating judgments: Given a set of yes or no questions or issues, the individuals' judgments are then aggregated separately, issue by issue. Applied in this way, the majority method does not guarantee the logical consistency of the set of judgments obtained. This fact has been the focus of critiques of the majority method and similar procedures. This paper focuses on another way of aggregating judgments. The main difference is that aggregation is made *en bloc* on all the issues at stake. The main consequence is that the majority method applied in this way does always guarantee the logical consistency of the collective judgments. Since it satisfies a large set of attractive properties, it should provide the basis for more positive assessment if applied using the proposed pooling approach than if used separately. The paper extends the analysis to the pooling supermajority and plurality rules, with similar results.

Key-words: judgment aggregation, pooling-, separating approach, majority-, special majority-, plurality voting.

JEL classification: D70; D71

1 Introduction

The discursive dilemma illustrates two features of the separating judgment approach on judgment aggregation (*JA*) that are useful to consider. First, the dilemma illustrates that separating majority voting (and similar judgment aggregation procedures such as the separating special majority and the separating plurality rules) may lead to a logically inconsistent collective output and therefore be unable to guarantee logically consistent collective judgments.[1] Given that logical consistency is deemed a fundamental property of collective judgments, this defect of majority voting and of similar judgment aggregation procedures has been their most frequently criticized feature and the main cause for the skepticism emerging from the separating *JA* approach about their feasibility as *JA* procedures.

[1] A majority is a subgroup of a group that contains more than half of the entire group. A special majority is a group larger than majority. A plurality is a subset having the largest number of individuals that may be less than majority.

Second, the discursive dilemma also illustrates that such inconsistencies are induced by the way the judgment aggregation is carried out under that approach. In such a framework, the collective judgments of the group are obtained by determining the collective judgment on each of the issues under consideration separately as a function of the individual judgments on it. This is the reason why we call this way of aggregating judgments 'separating judgment aggregation.' It differs from determining the collective judgments *en bloc* based on the comprehensive views of the individual group members on all the issues at stake. Let us call this latter kind of judgment aggregation 'pooling judgment aggregation.'

An imaginary example may help to clarify these points. Each year in the city of Pamplona there are celebrations for the day of Saint Fermín. The most internationally famous element of these celebrations is the bullfighting. With this in mind, a civil association known as Politeia called a meeting to propose and approve a declaration about such celebrations. They discussed six issues:

1) 'Bullfights are unjustified torture of the bulls' (proposition *a* or, if not, proposition 'not *a*')
2) 'Every unjustified torture of animals ought to be banned' (proposition *b* or, if not, proposition 'not *b*');
3) In particular, 'Bullfights should be abolished' (proposition *c* or, if not, proposition 'not *c*');
4) 'If they are not abolished, bullfights should be reformed, eliminating the cruelest parts' (proposition *d* or, if not, proposition 'not *d*');
5) 'The broadcasting of bullfights on television should be prohibited' (proposition *e* or, if not, proposition 'not *e*');
6) 'Children should be educated about this problem at school' (proposition *f* or, if not, proposition 'not *f*').

For simplicity, let us suppose that all members make a judgment on each of the six proposed statements. For this, the set of judgments made by each member can be represented by means of one of the sixty-four groups of the following form that we call complete judgment sets: $\{a, b, c, d, e, f\}$, $\{a, b, c, d, e, \text{not} f\}$, $\{a, \text{not } b, c, \text{not } d, e, f\}$, $\{\text{not } a, \text{not } b, c, d, \text{not } e, f\}$, $\{\text{not } a, \text{not } b, \text{not } c, \text{not } d, \text{not } e, \text{not } f\}$, etc.

Let us assume that the individual points of view are represented by the following three complete and consistent judgment sets:

Table 1.1

Judgment sets	**Support**
$\{a, b, c, d, e, f\}$	Subgroup A:1/3 of individuals
$\{\text{not } a, b, \text{not } c, d, e, f\}$	Subgroup B:1/3 of individuals
$\{a, \text{not } b, \text{not } c, d, \text{not } e, \text{not } f\}$	Subgroup C:1/3 of individuals

Under the separating aggregation approach, majority voting is carried out separately on each proposition-negation pair. Obviously, proposition *a* beats 'not *a*' because it is accepted by a majority of 2/3 of individuals. It is for the same reason that b beats 'not b', 'not c' beats c, e beats 'not e' and f beats 'not f,' whereas d beats 'not d' because d is unanimously accepted. Thus, the majority point of view is represented by the complete judgment set $\{a, b, \text{not } c, d, e, f\}$.

It should be noted that the judgment set $\{a, b, \text{not } c, d, e, f\}$ is not consistent, because propositions a and b together imply proposition c. This instance of the discursive dilemma illustrates that majority voting and similar judgment aggregation methods do not guarantee consistent collective judgment sets. As pointed out above, this flaw is the main reason for the impossibility and characterization results on separating judgment aggregation rules, and for the unfavorable assessment of majority and special majority rules.[2]

However, majority voting may be used in other ways as well. When groups arrive at collective judgments, they commonly use the rule of majority *en bloc* voting for texts dealing with diverse issues. For example, the United Nations Security Council frequently proposes resolutions dealing with various issues but approves them based on a single vote. When a parliament passes a law, a similar procedure is used. The passage of a law depends on a single vote on a larger proposal previously elaborated according to statutory procedure. Moreover, the public declarations of civil associations, whether scientific or cultural, often address a variety of issues. Such a declaration is made public if the group collectively passes the proposed text *en bloc*. Obviously, it would not be difficult to come up with many other examples. The point is that aggregating judgments *en bloc* by majority voting is a common way of aggregating judgments in practice.

The aim of this paper is to help to explain this common practice, which, under the separating approach to judgment aggregation, may seem paradoxical. In order to achieve that aim, I present a more general variant of the usual model of judgment aggregation, where judgments may be aggregated *en bloc*. It turns out that, in contrast to the usual separating *JA* approach, in the model presented in this paper the pooling majority rule, the pooling plurality rule, and any pooling special majority rule generate consistent collective judgment sets, provided that the individual judgment sets are consistent.

Let us refer back to Table 1.1 and take, for instance, the judgment set $\{b, d, e, f\}$. There is a majority of 2/3 of the individuals who accept it. In addition, since it is a subset of the individual judgment sets of subgroups A and B, it is a logically consistent judgment set. It is true that the set $\{b, d, e, f\}$ is not the only judgment set accepted by a majority of individuals. The following sets are also accepted by a majority: $\{b, d, e\}$, $\{b, d, f\}$, $\{d, e, f\}$, $\{a, d\}$, $\{b, d\}$, $\{b, e\}$, $\{b, f\}$, $\{\text{not } c, d\}$, $\{d, e\}$, $\{d, f\}$, $\{a\}$, $\{b\}$, $\{\text{not } c\}$, $\{d\}$, $\{e\}$ and $\{f\}$. The important thing is that all of these majority sets are logically consistent.

[2] As Dietrich and List (2008: 1-2) point out: 'From subsequent impossibility results we know that majority voting is not alone in its failure to ensure rational collective judgments on interconnected propositions, where rationality is understood as the conjunction of two requirements [consistency and completeness]…The generic finding is that dictatorships are the only proposition-wise aggregation functions generating consistent and complete judgments and satisfying some minimal conditions…This finding is broadly analogous to Arrow's theorem for preference aggregation.' For an overview of the main standard impossibility and characterization results, see List and Puppe (2009) or List (2009).

The example illustrates another feature of pooling judgment aggregation. Given the individual judgments assumed in Table 1.1, no complete judgment set achieves a majority and it would seem that the Politeia association cannot make public any declaration. Usually in such cases, groups seek a majority declaration even if this may not express a collective judgment on each of the issues at stake but a collective point of view on a smaller set of issues. For instance, the judgment set $\{b, d, e, f\}$ may be the collective judgment set chosen to represent the majority point of view of the assembly. The point here is that a choice takes place. The choice is necessary because, in such cases, pooling judgment aggregation rules generate more than one majority judgment set.

This feature of pooling judgment aggregation leads to a different view of the aggregation process and, consequently, to a different interpretation of the judgment aggregation model. In cases such as the Politeia association example, the whole process begins by aggregating from the individual points of view and ends with the choice of a majority judgment that is to be made public.[3] I propose to deal with such processes by describing them as consisting of two parts or stages. In the first stage, the group carries out the aggregation process, usually by obtaining several collective judgment sets. In the second stage, the group has to choose the judgment set that best expresses the group's point of view.

It is important to note that this choice may be far from obvious. For instance, some collective judgment sets may include judgments on a larger set of propositions, whereas others may be supported by more individuals. In addition, some judgments may be considered more important than others. This may be the case for judgments on propositions *a*, *b*, *c*, and *d* in our example. Furthermore, the decision may depend on the group and the context at hand, and if the decision has to be made collectively, it may face the well-known difficulties exemplified by the voting paradox.

In any case, it should be kept in mind that the model of pooling judgment aggregation presented below does not cover the choice stage. The model covers only the first stage, in which the judgment aggregation is carried out. [4]

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[3] In other situations, there may be no need of choosing one judgment set for expressing the group's point of view. Imagine, for example, that the directors of the Politeia association want merely to know whether the members of the association have a definite collective point of view on all of the points at stake, or only on some of them.

[4] Under the separating *JA* approach, there is no choice between a multiplicity of collective judgment sets because a judgment aggregation rule is defined in such a way that, in any situation, it generates only one collective judgment set (that may be not complete and may be empty).

Observe, however, that although the problem is not explicitly posed, this does not mean that it is completely non-existent. The separating way of using the majority rule in the Politeia association example gives the complete set $\{a, b, \text{not } c, d, e, f\}$ as the collective point of view. However, each subset of $\{a, b, \text{not } c, d, e, f\}$ also receives the support of a majority, as, for example, the subset $\{a, b, d, e, f\}$ or $\{a, b, d, f\}$. As neither of these two subsets presents any inconsistency, the question of what the ultimate reason is for the group having to choose the inconsistent group $\{a, b, \text{not } c, d, e, f\}$ as the collective point of view instead of $\{a, b, d, e, f\}$ or $\{a, b, d, f\}$ is raised. In this case, $\{a, b, \text{not } c, d, e, f\}$ has the advantage of being complete, but at the same time it is inconsistent, which works against it. In addition, at the time of choosing between judgment sets it should not always be a mandatory criterion that they encompass a set addressing a larger set of issues. There may be cases in which consistency or the degree of support are preferable criteria.

On the other hand, this does not depend on the appearance of inconsistencies. Imagine that the percentages of persons supporting each judgment set vary in the following manner: fifty-

I pointed out above that a salient feature of judgment aggregation illustrated by the Politeia association example is that there may be no collective judgment set that addresses all the points at issue. When this is the case, the usual requirement of full (logical) rationality is violated. Usually full rationality is conceived as the conjunction of logical consistency and completeness; in other words, a fully rational judgment aggregation procedure has to generate without fail a consistent collective judgment set that, in addition, addresses all of the issues at stake.

However, the requirement of completeness has been declining in importance compared to that of consistency. While it is true that in some kinds of situations completeness may be a compelling requirement, this is not always the case. As noted by Dietrich and List (2008), dropping the completeness requirement at the collective level is a frequently used means of avoiding the impossibility results obtained under the standard approach. Gärdenfors (2008) openly criticizes completeness as an unnatural requirement (see also García-Bermejo 2011). Dietrich and List (2007b and 2008) and Dokow and Holzman (2008) also investigate the consequences of eliminating the completeness requirement.[5] In a similar vein, the second section of this paper addresses the question of what logical consistency conditions may be reasonably required of pooling judgment aggregation procedures, setting aside the completeness restriction. I consider both consistency and completeness requirements in a later section (Section 4).

In Section 2, I introduce the model of pooling judgment aggregation, and show that pooling majority voting and a large class of similar judgment aggregation procedures satisfy relevant requirements of consistency.

In Section 3, I characterize pooling majority voting and pooling special majority rules. The section leads to a conclusion that appears to oppose those emerging from the separating *JA* framework on the feasibility of majority voting and special majority rules.

Section 4 addresses the question of what happens when the pooling *JA* rules are required to provide a complete collective output in addition to consistency. After some preliminary considerations, the section focuses on a weaker variant of the pooling plurality rule. Given that this rule always leads to a complete and consistent collective output, I show that the presented variant of the pooling plurality rule is the only one that satisfies not only the requirement of consistency

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one percent of the group members support the judgment set {*a*, *b*, *c*, *d*, *e*, *f*}, twenty-nine of them support the judgment set {*a*, *b*, *c*, *d*, *e*, not *f*}, and the remaining twenty percent support the judgment set {not *a*, *b*, not *c*, *d*, not *e*, not *f*}. In such a case, the complete judgment group which achieves a majority is {*a*, *b*, *c*, *d*, *e*, *f*}, with 51% of votes. Nonetheless, by eliminating the judgments on the proposition *f* we obtain the judgment set {*a*, *b*, *c*, *d*, *e*}, which achieves 80% of votes. I think that we cannot assume that any group will choose and make public the set {*a*, *b*, *c*, *d*, *e*, *f*} and not {*a*, *b*, *c*, *d*, *e*}.

These imaginary cases suggest that although judgments are aggregated separately, the problem of choosing a judgment set representative of the point of view of a collective persists, in spite of the fact that it is not mentioned in the separating *JA* approach.

[5] Dietrich and List (2007a) investigate necessary and sufficient conditions under which quota rules satisfy each of the requirements for full rationality (completeness, weak consistency, strong consistency, and deductive closure).

and completeness but also a relevant set of appealing properties. In addition, a similar characterization of the plurality rule is also provided.

2 A pooling approach to judgment aggregation

A judgment aggregation procedure generates collective judgments on the basis of the judgments made by the individuals in a group. Let $N=\{1, 2,.., n\}$ be a group of two or more persons.

2.1 Basic notations

For the sake of simplicity, let us adopt standard propositional logic as the logical framework (for a more general logical framework, see Dietrich 2007). Specifically, the propositions in the agenda are represented in a language $\boldsymbol{L}$ which contains (a) a given set of atomic propositions $a, b, c, \ldots$, and (b) compound propositions with the logical connectives : $\neg$(not), $\wedge$(and), $\vee$ (or), $\rightarrow$ (if, then), $\leftrightarrow$ (if and only if). Formally, $\boldsymbol{L}$ is the smallest set such that $a, b, c, \ldots \in \boldsymbol{L}$ and if $p, q \in \boldsymbol{L}$, then $\neg p$, $(p \wedge q)$, $(p \vee q)$, $(p \rightarrow q)$, $(p \leftrightarrow q) \in \boldsymbol{L}$.

In this logical framework, a *truth-value assignment* is a function assigning the value ‘true’ or ‘false’ (or ‘0’ and ‘1’ respectively) to each proposition in $\boldsymbol{L}$ in the standard way. A set of propositions $S \subseteq \boldsymbol{L}$ (a proposition $p \in \boldsymbol{L}$) (a) is *logically consistent* if there exists a truth-value assignment for which all the propositions in S are true (for which p is true); (b) is *logically inconsistent* otherwise; (c) S (proposition p) *logically entails* set S’ (proposition q) if for all truth-value assignments for which all the propositions in S are true (for which proposition p is true), all propositions in S’ are also true (q is also true); and (d) set S and set S’ (proposition p and proposition q) are *logically equivalent* if each of them logically entails the other.

The *agenda* X is a finite non-empty subset $X \subseteq L$ consisting of non-negated propositions and their negations. For the sake of simplicity, let us assume also that the agenda does not contain any twice-negated propositions (i.e., if a non-negated proposition p is in the agenda, then $\neg\neg p \notin X$). Let us call $\sim p$ the *complementary* proposition of p, and p the *complementary* proposition of $\sim p$, where $\sim p = \neg p$ if p is not itself a negated proposition, and $\sim p = q$ if p is the negated proposition $\neg q$. Let us suppose that the agenda X contains two or more propositions p and q such that p is not equivalent to q and is not equivalent to $\sim q$. Otherwise, the pooling approach cannot be distinguished from the separating one.

A sub-agenda $Y \subseteq X$ is any non-empty subset of the agenda consisting, like the agenda, of proposition-negation pairs $p, \neg p$. Notice that the agenda is also a sub-agenda. Given a sub-agenda Y, let Q^Y denote the set of all of the judgment sets A such that (a) if $p \in A$, then $p \in Y$, and (b) if $q \in Y$, then either $q \in A$ or $\neg q \in A$, or both. Note that Q^Y is an exhaustive set of sets that exclude each other, *i.e.*,

(1) $\bigvee_{X} q^A$ is a tautology, where $q^A = \wedge_{p \in A} p$, and (2) for any $A, A' \in Q^Y$, the set $\{A, A'\}$ is logically inconsistent. In addition, let Q^A denote Q^Y when $A \in Q^Y$.

The judgments that are made by individuals and by the group are represented by judgment sets. An individual or collective *judgment set* is a (possibly empty) subset of the agenda $A \subseteq X$.

A judgment set A is *complete* if there is no proposition p such that $p \in A$ and $\sim p \in A$; it is *weakly consistent* if, for any sub-agenda Y, the set $A \cap Y$ is a singleton; it is *strongly consistent* if it is *logically consistent*, that is, if there exists a truth-value assignment that assigns 'true' to each $q \in A$; it is *deductively closed* if, for every proposition p that is (logically) entailed by A, $p \in A$; and it is *fully consistent* if it is strongly consistent and deductively closed.

Notice that if a judgment set A is weakly consistent and deductively closed, then it is strongly consistent. Conversely, if A is strongly consistent and complete, then it is weakly consistent and deductively closed. However, a judgment set A may be deductively closed and inconsistent (for instance, the set $\{p, \neg p, (p \rightarrow q), q, \neg q\}$).

It is usually assumed that individuals make judgments on every proposition in the agenda and that, in addition, those judgments are consistent with each other. Thus, individual points of view are represented by complete and fully consistent judgment sets, and a *profile* of individual judgment sets is an n-tuple $(A_1, \ldots, A_n)$ of complete and fully consistent judgment sets, where A_i is the judgment set of individual i. The *universal domain* is the set of all the profiles of complete and fully consistent judgment sets.

2.2 Aggregating judgments *en bloc*

Let us introduce the model proposed for pooling *JA*. It differs from the separating *JA* approach in three main respects. First, whereas the input of an aggregation process in the pooling framework is, as in the separating case, a profile of individual judgment sets, the output is no longer a collective judgment set but a set of non-empty (collective) judgment sets. In consequence, aggregation procedures need to be represented by aggregation rules of a different kind.

Second, the majority method and similar aggregation procedures are used in a different way. Under the separating *JA* approach, these methods are applied separately to any proposition in the agenda, whereas in the pooling *JA* framework, they are applied to sets of such propositions.

Finally, the judgment aggregation stage may be complemented by an additional choice stage in which one (or several) of the collective judgment sets returned by the aggregation process should be chosen. However, this paper does not analyze the second stage, in which the familiar difficulties of preference aggregation may arise. The paper is confined to the first stage where judgment aggregation itself is carried out.

Take a set of non-empty judgment sets, J. Since the output of any *en bloc* aggregation process is any of such sets of non-empty judgment sets, let us consider their main properties.

As with other properties below, completeness can be understood in two ways. It may be imposed globally on the whole set of judgment sets J, or it may be required independently from the judgment sets within J. Let us say that a set of judgment sets J is *union-wise complete* iff $\cup_{A\in J}A$ is complete; that J is *element-wise complete* iff there is a complete $A\in J$; and that J is *contextually complete* iff for any sub-agenda Y, $Y\cap J\neq\varnothing$.

Notice that if a set of judgment sets J is contextually complete then it is element-wise complete, and if J is element-wise complete, then it is union-wise complete. The converse statements do not hold. Notice, in particular, that J may be union-wise complete while it is not element-wise complete.

In the pooling *JA* framework, completeness may demand too much. In any case, it requires significantly more than completeness in the separating *JA* approach. We will cover these issues below later. Let us now focus on the consistency conditions.

A set of judgment sets J is *union-wise weakly consistent* iff $\cup_{A\in J}A$ is weakly consistent; J is *element-wise weakly consistent* iff any $A\in J$ is weakly consistent.

A set of judgment sets J is *union-wise strongly consistent* iff $\cup_{A\in J}A$ is strongly consistent, and J is *element-wise strongly consistent* iff any $A\in J$ is strongly consistent. Obviously, union-wise strong consistency implies element-wise strong consistency and union-wise weak consistency. In addition, element-wise strong consistency implies element-wise weak consistency.

Notice that demanding deductive closure from any judgment set in J may be too strong a requirement because for some sub-agendas Y the judgment sets in Q^Y cannot meet such a demand. Therefore, we can demand from any judgment set A in J that if A implies another judgment set A' such that A logically entails A', then $A'\in J$. But this condition is equivalent to requiring that $\cup_{A\in J}A$ is deductively closed. So, let us say that a set of judgment sets J is *deductively closed* iff $\cup_{A\in J}A$ is deductively closed. In addition, let us say that J is *upper deductively closed*, if, for any $A\in J$ and any $A'\subseteq X$ such that A logically entails A", $A\cup A'\in J$; and that J is *lower deductively closed* if, for any $A\in J$ and any $A''\subseteq X$ such that $A''\subseteq A$, $A''\in J$. Obviously, if J is deductively closed, then it is upper and lower deductively closed.

It should be noted that for any set of judgment sets J,
(1) if J is union-wise (set-wise) weakly consistent and deductively closed, then it is union-wise (set-wise) strongly consistent;
(2) if J is union-wise complete and union-wise strongly consistent, then it is deductively closed.

Let us call a set of judgment sets J *union-wise fully consistent* iff it is union-wise strongly consistent and deductively closed; and let us call J *element-wise fully consistent* iff it is element-wise strongly consistent, and deductively closed.

The key element of the new model offered in this paper is the concept of *JA correspondence* (*JAC*). A *JAC* may be pooling or separating.

A *JA correspondence C* is a function that assigns to each profile $(A_1,\ldots, A_n)$ in its domain a set of non-empty judgment sets $C(A_1,\ldots, A_n)$.

A *separating JA correspondence C* is a *JAC* such that for any profile $(A_1,\ldots, A_n)$ in its domain, $A\in C(A_1,\ldots, A_n)$ is a singleton, *i.e.*, for any $A\in C(A_1,\ldots, A_n)$ there exists in the agenda a proposition p such that $\{p\}=A$.

A *pooling JA correspondence* is a non-separating *JAC*.

A separating *JA* method returns a separate collective judgment on each proposition of the agenda. It may or may not aggregate independently from the judgments on other propositions. If it does, the corresponding separating *JAC* satisfies the following independence property.

Proposition-wise independence. For any proposition $p\in X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C,
if $N_{\{p\}}=N^*_{\{p\}}$, then, $\{p\}\in C(A_1,\ldots, A_n)$ iff $\{p\}\in C(A^*_1,\ldots, A^*_n)$,
where $N_{\{p\}}=\{i\in N\text{: } \{p\}\subseteq A_i\}$.

Let us say that a separating *JAC* is a proposition-wise *JAC* if it satisfies that independence property.

Analogously, a pooling *JAC* may or may not satisfy the following set-wise independence property. If it does, let us say that it is a set-wise *JAC*.

Set-wise independence. For any judgment set $A\subseteq X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C,
if $N_A=N^*_A$, then, $A\in C(A_1,\ldots, A_n)$ iff $A\in C(A^*_1,\ldots, A^*_n)$, where $N_A=\{i\in N\text{: } A\subseteq A_i\}$.

The *JAC* concepts introduced above allow us to define the majority method and similar *JA* aggregation procedures in a way that changes with respect to the separating *JA* framework. The (*universal*) *pooling majority correspondence* is the function *SM* that for any profile $(A_1,\ldots, A_n)$ in the universal domain, $SM(A_1,\ldots, A_n)=\{A\subseteq X\text{: }|N_A|\geq\lceil (n+1)/2\rceil\}$, where $\lceil x\rceil$ denotes the smallest integer greater than or equal to x.

Likewise, given an integer α with $n\geq\alpha>\lceil (n+1)/2\rceil$, the (*universal*) *pooling α-special majority correspondence* is the function SM^{α} such that for any profile $(A_1,\ldots, A_n)$ in the universal domain, $SM^{\alpha}(A_1,\ldots, A_n)=\{A\subseteq X\text{: }|N_A|\geq\alpha\}$. And the (*universal*) *pooling plurality correspondence* is the function *SL* that for any profile $(A_1,\ldots, A_n)$ in the universal domain, $SL(A_1,\ldots, A_n)=\{A\subseteq X\text{: }|N_A|>|N_{A'}|$ for any $A'\in Q^A\}$.

It should be noted that in this paper, majority, special majority, and plurality rules are defined as universal pooling correspondence rules, that is, as *JACs* with the universal domain. Notice in addition that, in contrast to the pooling plurality correspondence, the pooling majority correspondence and any pooling α-special majority correspondence satisfy set-wise independence. Thus, they are set-wise *JACs*.

On the other hand, the (*universal*) *separating majority correspondence* is the function PM that for any profile $(A_1,\ldots, A_n)$ in the universal domain, $PM(A_1,\ldots, A_n)=\{\{p\}\subseteq X: |N_p| \geq \lceil (n+1)/2 \rceil\}$, where $N_p=\{i\in N: \{p\}\subseteq A_i\}$.

Analogously, given an integer α with $n\geq\alpha> \lceil (n+1)/2 \rceil$, the (*universal*) *separating α-special majority correspondence* is the function PM^{α} that for any profile $(A_1,\ldots, A_n)$ in the universal domain, $PM^{\alpha}(A_1,\ldots, A_n)=\{\{p\}\subseteq X: |N_p| \geq\alpha\}$. And the (*universal*) *separating plurality correspondence* is the function PL that for any profile $(A_1,\ldots, A_n)$ in the universal domain, $PL(A_1,\ldots, A_n)=\{\{p\}\subseteq X: |N_p| > |N_{\sim p}|\}$.

As in the pooling case and in contrast to the separating plurality correspondence, since the separating majority correspondence and any separating α-special majority correspondence satisfy proposition-wise independence, they are then proposition-wise *JACs*.

For simplicity, let us say that a *JAC* is union-wise complete, element-wise complete, contextually complete, etc. iff for any profile $(A_1,.., A_n)$ in its domain, the set of judgment sets $C(A_1,\ldots, A_n)$ is union-wise complete, element-wise complete, set-wise strongly complete, etc., respectively.

It can be easily verified that a pooling majority *JAC* for an agenda X and any group N is union-wise complete (union-wise weakly consistent, union-wise strongly consistent, deductively closed) iff the separating majority *JAC* for the agenda X and the group N is union-wise complete (union-wise weakly consistent, union-wise strongly consistent, deductively closed).[6]

2.3 The consistency question

The introduction of pooling *JAC*s raises the question of which rationality conditions may be reasonably imposed on these aggregation rules.

As pointed out in Section 1, completeness is no longer universally recognized in the literature as an unconditionally binding requirement in every situation. Additionally, element-wise completeness is an ever more demanding restriction, as we will see below. For these reasons, I consider the aforementioned question in two ways. First, I discuss which consistency restrictions can be reasonably imposed on the *JAC*s, leaving aside completeness, and afterwards in Section 4, I consider both sets of restrictions.

In contrast to completeness, logical consistency is generally considered an unconditionally binding requirement. Since I have mentioned several consistency conditions, the question is which ones have to be met unconditionally.

[6] By definition, $\{p\}\in SM(A_1,\ldots, A_n)$ iff $\{p\}\in PM(A_1,\ldots, A_n)$, $\{p\}\in SM^{\alpha}(A_1,\ldots, A_n)$ iff $\{p\}\in PM^{\alpha}(A_1,\ldots, A_n)$. Then = , and = . In addition, if $A\in SM(A_1,\ldots, A_n)$ and $p\in A$, then $\{p\}\in SM(A_1,\ldots, A_n)$. If $\{p\}\in SM(A_1,\ldots, A_n)$, then there is not any $A'\in SM(A_1,\ldots, A_n)$ such that $\sim p\in A'$. Thus, = . Analogously, = = . Therefore, = , and = .

In cases like the example of the Politeia association, the desired outcome in aggregating judgments *en bloc* is to obtain a collective judgment set that can adequately express the collective point of view of the group. If the aggregation stage generates more than one collective judgment set, the objective of the choice stage is to choose one of them. It therefore seems that there are three particularly desirable conditions on the collective set of judgment sets J resulting from the aggregation stage: 1) that J is element-wise strongly consistent; 2) that J is union-wise weakly consistent, in order to avoid collective ambiguities or collective contradictions with regard to any sub-agenda; 3) and that J is deductively closed, that is, if a judgment set A is accepted and it implies another judgment set A', then this latter judgment set should be also accepted. In short, J should be union-wise weakly consistent and element-wise fully consistent.

It is true that an element-wise fully consistent *JAC* may not be union-wise fully consistent. However, in cases like those depicted by our model, the final aim of the whole process is not to choose a set of judgment sets, but to choose one collective judgment set. This is why I maintain that in such cases, union-wise full consistency is a less urgent requirement than element-wise full consistency.

It turns out that in the pooling *JA* framework, a large class of *JAC*s satisfy element-wise strong consistency, namely the class of all the *JA* correspondences C such that if $A \in C(A_1,.., A_n)$, then $A \subseteq A_j$ for some individual j. In particular, and by contrast with the separating *JA* approach, the pooling majority and any pooling special majority correspondence satisfy element-wise strong consistency, element-wise and union-wise weak consistency, and deductive closure, as demonstrated by the following straightforward result. All this contrasts with the logical flaws that are usually emphasized in regard to the majority method under the separating *JA* approach. In addition, the pooling plurality correspondence also satisfies also element-wise strong consistency and element-wise and union-wise weak consistency. However, as far as deductive closure is concerned, it only satisfies upper deductive closure.[7]

Theorem 2.1.
(a) *The pooling majority correspondence*, *any pooling special majority correspondence, and the pooling plurality correspondence are*
(1) *union-wise weakly consistent and*, *therefore, union-wise and element-wise weakly consistent*,
(2) *element-wise strongly consistent*,
(3) *and upper deductively closed.*
(4) *In addition, the pooling majority correspondence and any pooling special majority*
correspondence are pooling deductively closed. They are, therefore, element-wise fully consistent.

Proof of Theorem 2.1.

[7] Imagine that, in a group with seven members, the judgment set $\{p, q, r\}$ is supported by three individuals, the judgment set $\{p, q, \neg r\}$ is supported by two individuals, and the judgment set $\{p, \neg q, \neg r\}$ is supported by the remaining two individuals. Then, in respect to the sub-agenda $\{p, \neg, r, \neg r\}$, the judgment set $\{p, r\}$ is supported by three individuals while the judgment set $\{p, \neg r\}$ is supported by four individuals. Thus, lower deductive closure is violated.

Let C be the pooling majority correspondence, or some pooling special majority or the pooling plurality rule, and let $A \in C(A_1,.., A_n)$ for any given profile $(A_1,.., A_n)$. Then $|N_A|>|N_{A'}|$ for every $A' \in Q^A$. Hence, C is union-wise weakly consistent. In addition, there is an individual j such that $A \subseteq A_j$. Thus, A is strongly consistent because A_i is strongly consistent. In regard to part 3, imagine that A implies A'. Then $N_A \subseteq N_{A \cup A'}$, and therefore, $|N_{A \cup A'}| \geq |N_A|$. In addition, for any $A^* \in Q^{A \cup A'}$ and any $A^o \in Q^A$, such that $A^o \subseteq Q$, $|N_{A^o}| \geq |N_{A^*}|$.
Thus, $|N_{A \cup A'}| \geq |N_A| > |N_{A^o}| \geq |N_{A^*}|$. Therefore, $|N_{A \cup A'}| > |N_{A^*}|$ for any $A^* \in Q^{A \cup A'}$ and $A \cup A' \in C(A_1,.., A_n)$. With regard to part 4, notice that if A implies A', then $N_A \subseteq N_{A'}$ and $|N_{A'}| \geq |N_A|$. Hence, if $A \in SM(A_1,.., A_n)$ then $A' \in SM(A_1,.., A_n)$, and if $A \in SM^{\alpha}(A_1,.., A_n)$, then $A' \in SM^{\alpha}(A_1,.., A_n)$. QED.

3 Characterization of the pooling majority and special majority correspondences

May's famous theorem states that the only one universal and decisive preference aggregation function that is anonymous, neutral, and positively responsive is the (binary or Condorcet's variant of the) majority method (May 1952).[8] In regard to *JA*, Dietrich and List's (2010b) Theorem 1 states that if the separating aggregation method is consistent, anonymous, and acceptance/rejection-neutral (a property close to systematicity) on a somehow restricted domain, then it is the separating majority method (restricted to that domain). In addition, it can easily be shown that the separating majority rule satisfies many other desirable properties. However, it is also known that if the agenda is not extremely simplified, then the separating majority rule fails to meet union-wise full consistency for some profiles of individual judgment sets, as the discursive dilemma illustrates (see List and Puppe 2009). This failure has been the main focus of critique lodged against the majority method in the *JA* literature.

In contrast, I have shown that the pooling majority correspondence is union-wise weakly consistent and element-wise fully consistent. Moreover, I have argued that in situations like that depicted by our model, element-wise fully consistency is more binding than union-wise strong consistency. Thus, the following question arises: Can the pooling majority correspondence be characterized similarly to that of May or to that of Dietrich and List? Can it be shown that pooling majority correspondence also satisfies a large set of desirable properties? If this is the case, then the majority method should be acknowledged as a feasible and relevant *JA* method for aggregating judgments *en bloc*. This assessment runs contrary to the usual assessment of the separating majority rule.

Theorem 3.1 below is a variant of May's theorem.[9] It not only states that the pooling majority correspondence satisfies some variants of the properties mentioned above in regard to May's theorem; it also states that the pooling majority correspondence is the only one that satisfies them. Consider the following properties:

[8] May calls the preference aggregation functions 'group decision functions.'
[9] I follow May's approach instead of that of Dietrich and List because, while the latter leads to restricting the domain of the majority rule, the former allows us to exploit the fact that this rule is element-wise fully consistent in the whole universal domain.

Proposition-wise anonymity. For any proposition $p\in X$ and any profiles $(A_1,\ldots,A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C that are permutations of each other, $\{p\}\in C(A_1,\ldots, A_n)$ iff $\{p\}\in C(A^*_1,\ldots, A^*_n)$.

Set-wise systematicity. For any judgment sets A, $A'\subseteq X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C,
if $N_A=N^*_{A'}$, then, $A\in C(A_1,\ldots, A_n)$ iff $A'\in C(A^*_1,\ldots, A^*_n)$.[10]

Notice that set-wise systematicity is a rather strong property because it states that sets that may be completely different in all respects and therefore different in size be treated the same if the subgroup of persons that support each of them is the same. It echoes the neutrality condition in social choice theory in the sense that any two pairs of
social states that are treated the same way by each individual should be treated the same way socially, independently of any other consideration. This feature lies at the heart of Sen's position against welfarism. In the next section, I introduce a weaker property, namely, that of contextual set-wise systematicity. However, the point here is that the pooling majority correspondence is set-wise systematic.

Proposition-wise positive responsiveness. Any *JA* correspondence C is *proposition-wise positively responsive*, iff for any $p\in X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in its domain, if $N_p\subset N^*_p$ and, in addition, $\{p\}\in C(A_1,\ldots, A_n)$ or $\{\sim p\}\notin C(A_1,\ldots, A_n)$,
then $\{p\}\in C(A^*_1,\ldots, A^*_n)$ and $\{\sim p\}\notin C(A^*_1,\ldots, A^*_n)$.

Theorem 3.1 *Let C be any universal JAC. C satisfies union-wise weak consistency, proposition-wise anonymity, set-wise systematicity, and proposition-wise positive responsiveness*, iff *C is the pooling majority correspondence*, *that is*, *for any profile* $(A_1,\ldots, A_n)$ *and any judgment set* $A\subseteq X$, $A\in C(A_1,\ldots, A_n)$ *iff* $|N_A|\geq \lceil (n+1)/2\rceil$.

Proof of Theorem 3.1. The 'then, if' part is straightforward. Therefore, let us focus on the 'if, then' part. Notice that by universality and set-wise systematicity, the statement 'for any profile $(A_1,\ldots, A_n)$ and any proposition p, $\{p\}\in C(A_1,\ldots, A_n)$ iff $|N_{\{p\}}|\geq\lceil (n+1)/2\rceil$' implies that for any profile $(A_1,\ldots, A_n)$ and any judgment set $A\subseteq X$, $A\in C(A_1,\ldots, A_n)$ iff $|N_A|\geq \geq\lceil (n+1)/2\rceil$. To verify this, let $(A^*_1,\ldots, A^*_n)$ be any profile in the domain of C such that for some proposition $p\in X$, $|N_A|=|N^*_{\{p\}}|$. By universality, there are such profiles in the domain of C. Imagine that the statement
'$\{p\}\in C(A^*_1,\ldots, A^*_n)$ iff $|N_{\{p\}}|\geq\lceil (n+1)/2\rceil$' holds. Given that $|N_A|=|N^*_{\{p\}}|$, systematicity implies then that $A\in C(A_1,\ldots, A_n)$ iff $|N_A|\geq\lceil (n+1)/2\rceil$.

[10] Obviously, set-wise systematicity implies proposition-wise systematicity:
Proposition-wise systematicity. For any p, $q\in X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C, if $N_{\{p\}}=N^*_{\{q\}}$, then, $\{p\}\in C(A_1,\ldots, A_n)$ iff $\{q\}\in C(A^*_1,\ldots, A^*_n)$.

Instead of proposition-wise systematicity, Dietrich and List (2010a) use the weaker property of unbiadness:
Proposition-wise unbiasedness. For any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of F and any proposition $p\in X$, if $N_p=N^*_{\sim p}$, then, $\{p\}\in C(A_1,.., A_n)$ iff $\{\sim p\}\in C(A^*_1,.., A^*_n)$.

Thus, to prove the theorem, it suffices to prove that for any profile $(A_1,\ldots, A_n)$ and any proposition p, $\{p\}\in C(A_1,\ldots, A_n)$ iff $|N_{\{p\}}|\geq\lceil(n+1)/2\rceil$. This is done in Claim 5.

Claim 1. If $|N_{\{p\}}|=|N_{\{\sim p\}}|$, then, $\{p\}\notin C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$.
Proof of Claim 1. If $|N_{\{p\}}|=|N_{\{\sim p\}}|$, then set-wise systematicity implies that $\{p\}\in C(A_1,\ldots, A_n)$ iff $\{\sim p\}\in C(A_1,\ldots, A_n)$. Thus, by union-wise weak consistency, $\{p\}\notin C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$. QED.

Claim 2. *If* $|N_{\{p\}}|>|N_{\{\sim p\}}|$, *then* $\{p\}\in C(A_1,\ldots, A_n)$.
Proof of Claim 3. Take from the universal domain a profile $(A'_1,\ldots, A'_n)$ such that $N'_{\{p\}}=N'_{\{\sim p\}}$ and $N'_{\{p\}}\subset N_{\{p\}}$. By Claim 1, $\{p\}\notin C(A'_1,\ldots, A'_n)$ and $\{\sim p\}\notin C(A'_1,\ldots, A'_n)$. Then by proposition-wise positive responsiveness, $\{p\}\in C(A_1,\ldots, A_n)$. QED.

Claim 3. *If* $|N_{\{p\}}|>|N_{\{\sim p\}}|$, *then* $\{\sim p\}\notin C(A_1,\ldots, A_n)$.
Proof of Claim 3. Assume that $|N_{\{p\}}|>|N_{\{\sim p\}}|$ and $\{\sim p\}\in C(A_1,\ldots, A_n)$ for a proof by contradiction. Take from the universal domain a profile $(A'_1,\ldots, A'_n)$ such that $N_{\{p\}}=N'_{\{\sim p\}}$ and $N_{\{\sim p\}}=N'_{\{p\}}$. Notice that by set-wise systematicity, $\{p\}\in C(A'_1,\ldots, A'_n)$. Take now another profile $(A''_1,\ldots, A''_n)$ from the universal domain such that $|N''_{\{p\}}|=|N'_{\{p\}}|$, $N''_{\{p\}}\subset N_{\{p\}}$, $N_{\{\sim p\}}\subset N''_{\{\sim p\}}$. Notice that the profile $(A''_1,\ldots, A''_n)$ is a permutation of the profile $(A'_1,\ldots, A'_n)$. Therefore, by anonymity $\{p\}\in C(A''_1,\ldots, A''_n)$. Then, given that $N''_{\{p\}}\subset N_{\{p\}}$, proposition-wise positive responsiveness implies that $\{p\}\in C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$, contradicting the assumption that $\{\sim p\}\in C(A_1,\ldots, A_n)$. QED.

Claim 4. $\{p\}\in C(A_1,\ldots, A_n)$ *and* $\{\sim p\}\notin C(A_1,\ldots, A_n)$, *iff* $|N_{\{p\}}|>|N_{\{\sim p\}}|$.
Proof of Claim 4. Notice that according to Claim 2 and 3, if $|N_{\{p\}}|>|N_{\sim p}|$, then $\{p\}\in C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$. Let us prove, then, that if $\{p\}\in C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$, then $|N_{\{p\}}|>|N_{\{\sim p\}}|$. Also by Claims 2 and 3, if $|N_{\{\sim p\}}|>|N_{\{p\}}|$, then $\{p\}\notin C(A_1,\ldots,A_n)$ and $\{\sim p\}\in C(A_1,\ldots, A_n)$. According to Claim 1, if $|N_{\{p\}}|=|N_{\sim p}|$, then $\{p\}\notin C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$. Therefore, if $\{p\}\in C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$, then $|N_{\{p\}}|>|N_{\sim p}|$. Hence, $\{p\}\in C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots,A_n)$, iff $|N_{\{p\}}|>|N_{\sim p}|$. Q.E.D.

Claim 5. For any profile $(A_1,\ldots, A_n)$ and any proposition p, $\{p\}\in C(A_1,\ldots, A_n)$ iff $|N_{\{p\}}|\geq\lceil(n+1)/2\rceil$.
Proof of Claim 5. By union-wise weak consistency and Claim 4, if $\{p\}\in C(A_1,\ldots, A_n)$, then $|N_{\{p\}}|\geq\lceil(n+1)/2\rceil$. On the other hand, Claim 3 implies that if $|N_{\{p\}}|\geq$ $\geq\lceil(n+1)/2\rceil$ then $\{p\}\in C(A_1,\ldots, A_n)$. QED.

A parallel result, namely Theorem 3.2, can be proven for any α-special majority correspondence by using an analogous argument. In this case, *proposition threshold need* and *proposition-wise α-positive responsiveness* are understood in the following way.

Proposition-wise threshold need. Let C be a *JAC* and let α be an integer such that $n\geq\alpha>\lceil(n+1)/2\rceil$. Let us say that *C needs the threshold* α if for any proposition $p\in X$ and any profile $(A_1,.., A_n)$ in the domain of C, if $\{p\}\in C(A_1,.., A_n)$ then $|N_p|\geq\alpha$.

Proposition-wise α-positive responsiveness. Any *JA* correspondence C is *proposition-wise α-positively responsive* iff for any proposition $p\in X$ and any profiles $(A_1,.., A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C,
(1) if $N_{\{p\}}\subset N^*_{\{p\}}$ and $\{p\}\in C(A_1,\ldots, A_n)$,
or (2) if $N_{\{p\}}\subset N^*_{\{p\}}$, in addition $\{p\}\notin C(A^*_1,\ldots, A^*_n)$ and $\{\sim p\}\notin C(A^*_1,\ldots, A^*_n)$, and also happens that $\alpha-|N_{\{p\}}|\leq 1$,
then $\{p\}\in C(A^*_1,\ldots, A^*_n)$ and $\{\sim p\}\notin C(A^*_1,\ldots, A^*_n)$.

Theorem 3.2. *Let C be any universal JAC. C satisfies union-wise weak consistency,*
proposition-wise anonymity, set-wise systematicity, proposition-wise threshold need,
and proposition-wise α-positive responsiveness, iff *C is the proposition-wise α-majority correspondence*, *that is*, *for any profile* $(A_1,\ldots, A_n)$ *and any judgment set* $A\subseteq X$, $A\in C(A_1,\ldots, A_n)$ *iff* $|N_A|\geq\alpha$.

The proof of Theorem 3.2, very similar to that of Theorem 3.1, can be found in the Appendix.

Notice that in contrast to what is usual under the separating *JA* approach, neither Theorem 3.1 nor Theorem 3.2 relies on the logical interconnections that may link the propositions in the agenda. They hold for any agenda.

In fact, the pooling majority correspondence and any pooling special majority correspondence satisfy stronger properties such as union-wise weak consistency and set-wise anonymity.

Set-wise anonymity. For any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C that are permutations of each other, $C(A_1,\ldots, A_n)=C(A^*_1,\ldots, A^*_n)$.

Regarding responsiveness, the pooling majority correspondence and any pooling special majority satisfy the following condition:

Set-wise non-negative responsiveness. For any judgment sets $A\subseteq X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C,
if $N_A\subseteq N^*_A$ and $A\in C(A_1,\ldots, A_n)$, then $A\in C(A^*_1,\ldots, A^*_n)$.[11]

In addition, the pooling majority correspondence satisfies the following condition of positive responsiveness:

[11] Set-wise non-negative responsiveness implies the following property:
Set-wise monotonicity. For any judgment set $A\subseteq X$, any $i\in N$ and any two i-variants $(A_1,.., A_i ,.., A_n)$, $(A_1,.., A'_i ,.., A_n)$ in the domain of C,
if $A \not\subseteq A_i$, $A\subseteq A'_i$ and $A\in C(A_1,.., A_i ,.., A_n)$, then $A\in C(A_1,.., A'_i,.., A_n)$,
where two profiles $(A_1,.., A_i ,.., A_n)$, $(A'_1,.., A'_i ,.., A'_n)$ are i-variants of each other if, for any $j\neq i$, $A_j=A'_j$.

Set-wise majority positive responsiveness. C is *set-wise majority positively responsive*, iff for any judgment sets $A \subseteq X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C, if 1) $N_A \subset N^*_A$, and 2) $\sum_{A' \in Q^A, A' \neq A} |N_{A'}| \leq \lceil (n+1)/2 \rceil$, then $A \in C(A^*_1,\ldots, A^*_n)$ and $A' \notin C(A^*_1,\ldots, A^*_n)$ for any $A' \in Q^A$ such that $A' \neq A$.

In turn, the pooling majority correspondence satisfies this one:

Set-wise supermajority positive responsiveness. C is *set-wise supermajority positively responsive*, iff for any judgment sets $A \subseteq X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C, if 1) $N_A \subset N^*_A$; and 2) $\sum_{A' \in Q^A, A' \neq A} |N_{A'}| \leq |N| - \varkappa$, then $A \in C(A^*_1,\ldots, A^*_n)$ and $A' \notin C(A^*_1,\ldots, A^*_n)$ for any $A' \in Q^A$ such that $A' \neq A$.

The conditions in Arrow's theorem (*unanimity*, *independence*, and *non-dictatorship*) are also known properties of majority voting and of any special majority rule. Set-wise independence has been introduced above. Let us adapt the other two properties to our framework.

Set-wise unanimity. For any profile $(A_1,\ldots, A_n)$ in the domain of C and any judgment set $A \subseteq X$, if, for every individual $i \in N$, $A \subseteq A_i$, then $A \in C(A_1,\ldots, A_n)$.

Non-set-wise dictatorship. There exists no $j \in N$ such that for any profile $(A_1,\ldots, A_n)$ in the domain of C, and any judgment set $A \subseteq X$, if $A \subseteq A_j$ then $A \in C(A_1,\ldots, A_n)$.

At times in the literature on *JA*, this property is reinforced by the following one.

Non-set-wise local dictatorship. There exists no $i \in N$ and no judgment set $A \subseteq X$ such that for any profile $(A_1,\ldots, A_n)$ in the domain of C, if $A \subseteq A_i$ then $A \in C(A_1,\ldots, A_n)$.

Notice that taking $A=A^*$, set-wise systematicity implies set-wise independence. Similarly, it is apparent that set-wise anonymity implies non-set-wise dictatorship and non-set-wise local dictatorship.

Translating Gibbard's concept of an oligarchy, the following condition of non-oligarchy can be proposed.[12] It is apparent that this property is satisfied by the

[12] In the literature on *JA* another weaker concept of oligarchy is used (see, for instance, Dietrich and List 2008). In particular, the requirement that any oligarch has veto power has been eliminated.

pooling majority correspondence and by any pooling special majority correspondence except the unanimity correspondence.

Non-set-wise oligarchy (in Gibbard's sense). There is no non-empty group $G \subseteq N$ such that for any profile $(A_1,\ldots, A_n)$ in the domain of C and any judgment set A,
(a) if $A \subseteq \bigcap_{i\in} A_i$, then $A \in C(A_1,\ldots, A_n)$,
and (b) for any $i \in G$, if $A' \subseteq A_i$, then $A'' \notin C(A_1,\ldots, A_n)$ for all $A'' \in Q^A$ such that $A' \neq A''$.

Furthermore, adapting the third part of the proof of Theorem 1 in Dietrich and List (2007c: 28), it can be shown that any universal, set-wise independent and set-wise monotonic *JAC* is non-set-wise manipulable, in the sense introduced by those authors. Adapting the notion of manipulability introduced by them, let us say that one judgment set A agrees with a set of judgment sets J on a judgment set $A' \subseteq X$, if (a) $A' \subseteq A$ and $A' \in J$, or (b) not $A' \subseteq A$, and $A' \notin J$; A disagrees with J on A otherwise.

Correspondence C is set-wise manipulable at the profile $(A_1,.., A_i,.., A_n)$ by individual i on the judgment set $A \subseteq X$ (in Dietrich-List sense),
if A_i disagrees with $C(A_1,.., A_i,.., A_n)$ on A,
but A_i agrees with $C(A_1,.., A^*_i,.., A_n)$ on A for some i-variant $(A_1,.., A^*_i,.., A_n)$.

Summing up. According to Theorems 3.1 and 3.2,
any universal pooling *JA* correspondence C satisfies
(1) union-wise weak consistency,
(2) proposition-wise anonymity,
(3) set-wise systematicity,
and (4) proposition-wise positive responsiveness (proposition-wise threshold need, and proposition-wise α-positive responsiveness),
iff C is the pooling majority correspondence (C is the pooling α-special majority),

In addition, it should be noted that the pooling majority correspondence (any pooling α-special majority correspondence) also satisfies:
(1) union-wise and element-wise weak consistency,
(2) element-wise strong consistency,
(3) deductive closure,
(3) set-wise anonymity,
(5) set-wise majority positive responsiveness, set-wise non-negative responsiveness, and set-wise monotonicity (set-wise supermajority positive responsiveness, set-wise non-negative responsiveness, and set-wise monotonicity),
(6) set-wise unanimity,
(7) set-wise independence,
(8) non-set-wise dictatorship and non-set-wise local dictatorship,
and (9) non-set-wise manipulability or set-wise strategy-proofness.

The pooling majority correspondence and any pooling α-special majority correspondence with $\alpha < |N|$ satisfy also non-set-wise oligarchy (in Gibbard's sense).

This summary illustrates an especially relevant consequence of accepting element-wise strong consistency as the main logical restriction on *JAC*s. The pooling majority correspondence is the only *JAC* that, in addition to element-wise strong consistency and deductive closure, and except union-wise and element-wise completeness, satisfies all the properties that are usually presented as desirable conditions on the *JA* rules. Thus, this summary may explain why in practice the majority method is frequently used for aggregating judgments *en bloc*. In any case, Theorem 3.1, Theorem 3.2 and the above summary may provide the basis for a favorable assessment of the feasibility of the pooling majority and pooling special majority rules in contrast to those emerging from the separating *JA* approach.

4 Demanding completeness

In the preceding sections, completeness has been not required from the *JA* rules. Let us now address the case in which completeness is imposed on a *JAC* in addition to consistency.

4.1 Completeness: a restrictive condition

The pooling majority correspondence is neither contextually complete, nor element-wise complete, nor even union-wise complete. Thus, a new impossibility corollary may be derived from Theorem 3.1.

Corollary 4.1.
(a) *There is no union-wise complete*, *union-wise weakly consistent JAC that satisfies proposition-wise anonymity, set-wise systematicity, and proposition-wise positive responsiveness*.
(b) *If* $|N|$ *is odd*, *then there is no element-wise complete*, *union-wise weakly consistent JAC that satisfies proposition-wise anonymity, set-wise systematicity, and proposition-wise positive responsiveness*.

Requiring element-wise or contextual completeness from the separating majority correspondence may lack sense because *C* includes only singletons. Therefore, the relevant completeness condition in this case is union-wise completeness.

By contrast, contextual and element-wise completeness are at least as relevant as union-wise completeness in the case of the pooling majority correspondence. Notice also that since the pooling majority and any special majority correspondences are lower deductively closed, they are contextually complete if and only if they are element-wise complete.

It has been shown in Section 2 that the separating majority correspondence is union-wise complete if and only if the pooling majority correspondence is also union-wise complete. However, generating a element-wise complete collective outcome by the majority method in the pooling *JA* framework is a more difficult task than generating a union-wise complete collective outcome in the separating or in the pooling case, provided that there are in the agenda two or more propositions *p* and *q* such that *p* is neither equivalent to *q* nor to *~q*.

Having an odd number of persons in the group is a sufficient condition for obtaining a union-wise complete collective set of judgment sets with the separating majority correspondence. This is because, in order to obtain the complete collective judgment set, it suffices that the group chooses one of the two propositions contained in each of the proposition-negation sets $\{p, \sim p\} \subseteq X$. By contrast, in the case of the pooling majority correspondence, that condition is no longer sufficient for element-wise completeness. Notice that if the agenda X contains two or more propositions p and q such that p is not equivalent either to q or to $\sim q$, then Q^X contains more than two propositions. Let us compare, for instance, the agenda $X'=\{p, \sim p\}$ with the agenda $X''=\{p, \sim p, q, \sim q\}$, where p implies q but q does not imply p. Notice that X'' is the slightest departure from X'. However, $Q^{X''}$ contains three consistent judgment sets: $\{p, q\}$, $\{p, \sim q\}$, and $\{\sim p, \sim q\}$ while $Q^{X'}$ contains only two: $\{p\}$ and $\{\sim p\}$. Obviously, as the number of non-equivalent propositions in an agenda X increases, the number of consistent complete judgment sets also increases and, consequently, the probability of obtaining a complete collective output decreases.

It should be noted, in addition, that for a class of *JAC*s that contain the pooling majority correspondence and any pooling special majority correspondence, element-wise completeness implies element-wise full consistency for any profile of individual judgment sets.

Theorem 4.1 *Let C be any union-wise weakly consistent and lower deductively closed JAC, and let* $(A_1,.., A_n)$ *any profile of individual judgment sets. In addition, let C be such that for any* $A \in C(A_1,.., A_n)$ *there is an individual such that* $A \subseteq A_j$.
(1) *If* $C(A_1,.., A_n)$ *is element-wise complete, then it is element-wise strongly consistent.*
(2) *Therefore, if* $C(A_1,.., A_n)$ *is element-wise complete and deductively closed, then it is element-wise fully consistent.*

Proof of Theorem 4.1.
If C is element-wise complete, then there is a $A \in C(A_1,.., A_n)$ that is complete and such that for an individual $A=A_j$. Thus, A is element-wise fully consistent and, by union-wise weak consistency, is the only complete set in $C(A_1,.., A_n)$. Let Y be any sub-agenda. There is a judgment set A' in Q^Y such that $A' \subseteq A$. Therefore, A' is strongly consistent. By lower deductive closure, $A' \in C(A_1,.., A_n)$. By union-wise weak consistency, A' is the only judgment set in $Q^{A'} \cap C(A_1,.., A_n)$. Hence, $C(A_1,.., A_n)$ is element-wise strongly consistent.
QED.

4.2 The pooling weak plurality correspondence

While the compulsory strength of a general and unconditional requirement of completeness in *JA* procedures has become more dubious than before, a lack of completeness may cause serious problems in some situations where completeness may become highly desirable or even compelling. One feasible method for overcoming this flaw involves two features. On the one hand, the majority requirement is lowered, requiring only that the chosen judgment sets are supported by a subgroup at least as large as any other subgroup that supports any of the alternative judgment sets. On the other hand, a tie-breaking method is used when necessary. Let us represent the aggregation stage of this procedure by the following *JAC*.

The *pooling weak plurality correspondence* is the *JAC* that assigns to profile $(A_1,\ldots, A_n)$ in the universal domain the following set of judgment sets $SW(A_1,\ldots, A_n)=\{A\subseteq X: |N_A|\geq|N_{A'}| \text{ for all } A'\in Q^A\}$.

It is true that no tie-breaking method is included in this *JA* rule. However, this rule only models the aggregation stage, like the other *JACs* that we have introduced in this paper. The tie-breaking method that we should add may be considered and analyzed as a part of the subsequent choice stage.

The pooling weak plurality correspondence is element-wise complete at the expense of satisfying neither union-wise or union-wise weak consistency nor lower deductive closure. However, it always satisfies element-wise strong consistency. Notice in addition that like the pooling majority and special majority correspondences it satisfies set-wise anonymity. It is true that it does not satisfy set-wise systematicity or set-wise majority positive responsiveness. However, it can be characterized by the following weakened variants of these properties, jointly with element-wise completeness and set-wise anonymity.

Contextual set-wise systematicity. For any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain, and any sub-agenda $Y\subseteq X$, if there is a one-to-one mapping g of Q^Y onto itself such that $N_A=N^*_{g(A)}$, then, for all $A\in Q^Y$, $A\in C(A_1,\ldots, A_n)$ iff $g(A)\in C(A^*_1,\ldots, A^*_n)$.

Contextual set-wise systematicity may be a more appealing property than set-wise systematicity for some aggregation problems. For instance, it is satisfied by a common parliamentary practice that violates set-wise systematicity. In such cases, higher acceptance thresholds of votes are required on certain issues that are considered more important than others.

Given a profile $(A_1,\ldots, A_n)$ and a judgment set, let k=Given a profile $(A_1,\ldots, A_n)$ and a judgment set, let k=

Set-wise weak plurality positive responsiveness.
C is *set-wise weak plurality positively responsive*, iff
for any judgment sets $A\subseteq X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C, if 1) $N_A\subset N^*_A$; 2) for any $A'\in Q^A$ such that $A'\neq A$, $|N_{A'}|\geq|N^*_{A'}|$, then
(a) if $(|N^*_A|-|N_A|)\geq k$, then $A\in C(A^*_1,\ldots, A^*_n)$,
and (b) if $A\in C(A_1,\ldots, A_n)$ or $(|N_{A^*}|-|N_A|)>k$, then $A\in C(A^*_1,\ldots, A^*_n)$ and $A''\notin C(A^*_1,\ldots,A^*_n)$ for any $A''\in Q^A$ such that $A''\neq A$.

Since the proof of the following theorem is very similar to that of Theorem 3.1, it is included in the Appendix.

Theorem 4.2 *Any universal C satisfies contextual completeness, set-wise anonymity, contextual systematicity, and set-wise weak plurality positive responsiveness*, *iff* , *C is the pooling weak plurality correspondence*, *that is*, *for any profile* $(A_1,\ldots, A_n)$ *and any judgment set* $A \subseteq X$, $A \in C(A_1,\ldots, A_n)$ *iff* $|N_A| \geq |N_{A'}|$ *for all* $A' \in Q^{A'}$.

Given that pooling weak plurality does not satisfy union-wise weak consistency, the following corollary follows from Theorem 4.2.

Corollary 4.2. *There is no universal JAC that satisfies contextual completeness*, *union-wise weak consistency*, *set-wise anonymity, contextual set-wise systematicity*, *and set-wise weak plurality positive responsiveness*.

As far as the conditions required in Arrow's theorem are concerned, pooling weak plurality satisfies set-wise unanimity; since it is set-wise anonymous it is not set-wise dictatorial, not set-wise locally dictatorial, and not set-wise oligarchical; in addition, contextual set-wise systematicity implies contextual set-wise independence.

Contextual set-wise independence. For any sub-agenda $Y \subseteq X$, if $N_A = N^*_A$ for any judgment set $A \in Q^Y$, then for any $A' \in Q^Y$, $A' \in C(A_1,\ldots, A_n)$ iff $A' \in C(A^*_1,\ldots, A^*_n)$.

Another two nice properties of the *pooling weak plurality* are the following.

Efficiency. A *JA* correspondence *C* is efficient iff for any profile $(A_1,.., A_n)$ in its domain and any sub-agenda *Y*, $Q^Y \cap C(A_1,.., A_n) \neq \varnothing$.

Minimal efficiency. A *JA* correspondence *C* is minimally efficient iff for any profile $(A_1,.., A_n)$ in its domain, $C(A_1,.., A_n) \neq \varnothing$.

Obviously, any efficient *JAC* is minimally efficient. Notice, in addition, that the pooling and the separating majority correspondences, as well as the pooling and separating special majority correspondences are not minimally efficient. If $|N|$ is odd, then the pooling and the separating majority correspondences are minimally efficient but not necessarily efficient. On the other hand, for any pooling *JA* correspondence *C*, 1) if *C* is union-wise complete, then it is minimally efficient, and 2) *C* is efficient iff it is element-wise complete.

However, plurality achieves element-wise completeness at the cost of violating the following property.

Non-majority rejection. A *JA* correspondence *C* satisfies this property if, for any profile $(A_1,.., A_n)$ in its domain and any judgment set $A \in C(A_1,.., A_n)$ there is no majority that rejects *A*, that is, there is no group $G \subseteq N$, such that 1) $|G| \geq \lceil (n+1)/2 \rceil$, and 2) for any person $j \in G$, $A \cup A_j$ is logically inconsistent.

For some *JA* problems, such as the passage of a law in parliament, this condition may not be relevant. But in cases such as those illustrated by the Politeia association example, it seems clearly plausible.

4.3 The pooling plurality correspondence

We can also define the binary relation '*as least as efficient as*' between pooling and separating *JAC*s in the following way. The *JA* aggregation correspondence C is *as least as efficient as* the *JA* aggregation correspondence C' iff
(a) The domain of C' is a (non-necessary proper) subset of the domain of C;
(b) for any profile $(A_1,\ldots, A_n)$ in the domain of C, any sub-agenda $Y\subseteq X$,
if $Q^Y\cap C'(A_1,.., A_n)\neq\varnothing$, then $Q^Y\cap C(A_1,.., A_n)\neq\varnothing$.

As usual, we say that the *JA* correspondence C is *more efficient than* the *JA* correspondence C', iff C is as least as efficient as C' but C' is not as least as efficient as C; and C is *equally efficient as* C', iff C is as least as efficient as C' and C' is as least as efficient as C.

Let say, in addition, that the *JA* correspondence C is *an extension of* the *JA* correspondence C' if, for any sub-agenda $Y\subseteq X$, such that
$Q^Y\cap C'(A_1,.., A_n)\neq\varnothing$, it happens that $C(A_1,.., A_n)= C'(A_1,.., A_n)$.

Notice that if $|N|>5$, (1) the pooling majority correspondence is an extension of any pooling majority correspondence and is more efficient than it; (2) the pooling plurality correspondence is an extension of the pooling majority correspondence and is more efficient than it; and analogously, (3) the pooling weak plurality correspondence is a more efficient extension of the pooling plurality correspondence.

The pooling weak plurality correspondence is element-wise complete, but it is not union-wise weakly consistent. However, it may be the case that gaining completeness at the expense of weak consistency is considered unnecessary. So, the pooling plurality correspondence, which is more efficient than the pooling majority correspondence may be sometimes considered a suitable option.

Obviously, the pooling plurality correspondence satisfies the following condition.

Contextual semi-completeness.
For any profile $(A_1,\ldots, A_n)$ in the universal domain, and any sub-agenda $Y\subseteq X$,
if there is a judgment set $A\in Q^Y$ such that $|N_A|>|N_{A'}|$ for any $A'\in Q^Y$, then there is a judgment set $A''\in Q^Y$ such that $A''\in C(A_1,\ldots, A_n)$.

Consider also the following variant of positive responsiveness.

Set-wise plurality positive responsiveness.
C is *set-wise plurality positively responsive*, iff
for any judgment sets $A\subseteq X$ and any profiles $(A_1,\ldots, A_n)$, $(A^*_1,\ldots, A^*_n)$ in the domain of C, if 1) $N_A\subset N^*_A$; and 2) for any $A'\in Q^A$ such that $A'\neq A$, $|N_{A'}|\geq|N^*_{A'}|$, then,
if $(|N^*_A|-|N_A|)>k$ or $A\in C(A_1,\ldots, A_n)$, then $A\in C(A^*_1,\ldots, A^*_n)$ and $A''\notin C(A^*_1,\ldots, A^*_n)$ for any $A''\in Q^A$ such that $A''\neq A$, where k=
for any $A''\in Q^A$ such that $A''\neq A$, where k=
It turns out that substituting contextual semi-completeness for contextual completeness, substituting set-wise plurality positive responsiveness for set-wise

weak plurality positive responsiveness, and adding union-wise weak consistency, the following characterization of the pooling plurality rule can be obtained.

Theorem 4.3. *For any universal C, C satisfies contextual semi-completeness, union-wise weak consistency, set-wise anonymity, contextual systematicity and set-wise plurality positive responsiveness, iff C is the pooling plurality correspondence.*

The proof of Theorem 4.3 is entirely analogous to that of Theorem 4.2. It is therefore included in the Appendix.

The following is a straightforward corollary of Theorem 4.3.

Corollary 4.3. *There is no universal JAC that satisfies contextual completeness, union-wise weak consistency, set-wise anonymity, contextual set-wise systematicity, and set-wise plurality positive responsiveness.*

Let us conclude this section summarizing the properties of the pooling weak plurality and of the pooling plurality correspondences.

Theorems 4.2 and 4.3 state that any universal pooling *JA* correspondence satisfies
(1) contextual completeness (contextual semi-completeness and union-wise weak consistency),
(2) set-wise anonymity,
(3) contextual systematicity,
(4) and set-wise weak-plurality positive responsiveness (set-wise plurality positive responsiveness),
iff C is the pooling weak plurality (the pooling plurality) correspondence.

In addition, the pooling weak plurality (the pooling plurality) correspondence also satisfies:
(1) element-wise strong consistency,
(2) set-wise unanimity,
(3) contextual set-wise independence,
(4) non-set-wise dictatorship, non-set-wise local dictatorship and non-set-wise oligarchy,
and (5) efficiency (not-minimal efficiency).

5 Concluding remarks

The pooling *JA* approach ensures the element-wise full consistency for a large class of *JA* procedures, namely, all those correspondences such that if $A \in C(A_1,.., A_n)$, then $A \subseteq A_j$ for some individual $j \in N$. In particular, (universal) pooling majority and (universal) pooling special majority rules are element-wise fully consistent. In addition, I have argued that element-wise full consistency is more binding for *JAC*s than union-wise full consistency.

Further, (universal) pooling majority voting is the only (universal) pooling *JA* method that satisfies a large and known set of generally desirable properties on the aggregation process, likewise each (universal) pooling special majority rule is the only (universal) pooling *JA* method that satisfies another different but similar set of relevant properties. Therefore, in this paper I have maintained that the (universal) pooling special majority rules and especially the (universal) pooling majority rule may be considered as salient and feasible pooling *JA* procedures, in contrast to the unfavorable assessment of (universal) separating majority and of (universal) separating special majority rules. I have also claimed that the analysis of pooling majority voting carried out in Sections 2 and 3 can help to explain the common practice of aggregating individual judgments *en bloc* by majority voting.

Pooling majority and special majority rules sacrifice completeness in order to satisfy a large number of desirable properties. While completeness is no longer universally recognized as an unconditionally binding requirement on *JA* procedures in all situations, it is nevertheless a desirable property and may be a compelling requirement to address some *JA* problems. Thus, in Section 4, I described a fully complete (universal) pooling *JA* method, namely (universal) pooling weak plurality correspondence. While *JAC* does not satisfy some consistency conditions such as union-wise weak consistency and deductive closure, it is element-wise strongly consistent. Thus, it satisfies the basic consistency requirement. In addition, it is upper deductively closed, and if a tie-breaking procedure is used, the collective output would be union-wise weakly consistent. It is true that the pooling weak plurality correspondence does not satisfy some other properties of majority voting, such as set-wise systematicity and set-wise majority positive responsiveness. However, it satisfies weakened versions thereof. It has also been shown that the (universal) pooling weak plurality correspondence is the only (universal) element-wise complete and element-wise strongly consistent *JAC* that satisfies set-wise anonymity and weakened versions of set-wise systematicity and set-wise majority positive responsiveness.

Similarly, the (universal) pooling plurality correspondence is the only (universal) union-wise weakly consistent, element-wise strongly consistent, and upper deductively closed *JAC* that satisfies set-wise anonymity, a weakened version of set-wise systematicity, set-wise plurality positive responsiveness, and in addition, a weakened version of completeness. Thus, as in the case of majority and special majority correspondences, the assessment of (universal) weak plurality and (universal) plurality stemming from the pooling approach should be more favorable than that of their separating variants.

However, weak plurality and plurality are less satisfactory methods of aggregation than majority and special majorities. Among other considerations, they do not satisfy the non-majority rejection condition. Thus, in such *JA* aggregation problems in which completeness is highly desirable or not at all expendable, the pooling approach may not be of great help. It should be noted, however, that the separating approach may not be any more helpful. Since the agenda X contains two or more propositions p and q such that p is not equivalent to q and is also not equivalent to $\sim q$, the pooling majority correspondence, the pooling α-special majority correspondence, the pooling plurality correspondence, and the pooling weak plurality correspondence are each a more efficient extension of the corresponding separating variant.

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A Appendix: Additional proofs.

Proof of Theorem 3.2.

Like with Theorem 3.1, we focus on the 'if, then' part. Notice, in addition, that by universality and set-wise systematicity, the statement 'for any profile $(A_1,\ldots, A_n)$ and any proposition p, $\{p\}\in C(A_1,\ldots, A_n)$ iff $|N_{\{p\}}|\geq\alpha>\lceil(n+1)/2\rceil$' implies that for any profile $(A_1,\ldots, A_n)$ and any judgment set $A\subseteq X$, $A\in C(A_1,\ldots, A_n)$ iff $|N_A|\geq\alpha>$ $>\lceil(n+1)/2\rceil$. Therefore, in order to prove the theorem it suffices to prove that for any profile $(A_1,\ldots,A_n)$ and any proposition p, $\{p\}\in C(A_1,\ldots, A_n)$ iff $|N_{\{p\}}|\geq\alpha>$ $>\lceil(n+1)/2\rceil$. This is done in Claim 5.

Claim 1. If $|N_{\{p\}}|<\alpha$ and $|N_{\{\sim p\}}|<\alpha$, then $\{p\}\notin C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$.

Proof of Claim 1. Suppose for a proof by contradiction that $\{p\}\in C(A_1,\ldots, A_n)$ or $\{\sim p\}\in C(A_1,\ldots, A_n)$. Proposition-wise threshold need implies that $|N_{\{p\}}|\geq\alpha$ or $|N_{\{\sim p\}}|\geq\alpha$, contradicting the assumption. QED.

Claim 2. *If* $|N_{\{p\}}|\geq\alpha$, *then* $\{p\}\in C(A_1,\ldots, A_n)$.
Proof of Claim 2. Take from the universal domain a profile $(A'_1,\ldots, A'_n)$ such that $N'_{\{p\}}\subset N_{\{p\}}$, $N_{\{\sim p\}}\subset N'_{\{\sim p\}}$, and $\alpha\text{-}|N'_{\{p\}}|=1$. Then $|N'_{\{p\}}|<\alpha$, and $|N'_{\{\sim p\}}|<\alpha$. By Claim 1, $\{p\}\notin C(A'_1,\ldots, A'_n)$ and $\{\sim p\}\notin C(A'_1,\ldots, A'_n)$. But then, proposition-wise α-positive responsiveness implies that $\{p\}\in C(A_1,\ldots, A_n)$. QED.

Claim 3. *If* $|N_{\{\sim p\}}|\leq|N|\text{-}\alpha$, *then* $\{\sim p\}\notin C(A_1,\ldots, A_n)$.
Proof of Claim 3. Assume that $|N_{\{\sim p\}}|\leq|N|\text{-}\alpha$ and $\{\sim p\}\in C(A_1,\ldots, A_n)$ for a proof by contradiction. Since $\alpha>\lceil(n+1)/2\rceil$, then $|N_{\{p\}}|\geq\alpha$, and $|N_{\{p\}}|>|N_{\{\sim p\}}|$. Take from the universal domain a profile $(A'_1,\ldots, A'_n)$ such that $N_{\{p\}}=N'_{\{\sim p\}}$ and $N_{\{\sim p\}}=N'_{\{p\}}$. Notice that by set-wise systematicy, $\{p\}\in C(A'_1,\ldots, A'_n)$. Take now another profile $(A''_1,\ldots, A''_n)$ from the universal domain such that $|N''_{\{p\}}|=|N'_{\{p\}}|$, $N''_{\{p\}}\subset N_{\{p\}}$, $N_{\{\sim p\}}\subset N''_{\{\sim p\}}$. Notice that the profile $(A''_1,\ldots, A''_n)$ is a permutation of the profile $(A'_1,\ldots, A'_n)$. Therefore, by anonymity $\{p\}\in C(A''_1,\ldots, A''_n)$. Then, given that $N''_{\{p\}}\subset N_{\{p\}}$, proposition-wise positive responsiveness implies that $\{p\}\in C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$, contradicting the assumption that $\{\sim p\}\in C(A_1,\ldots, A_n)$. QED.

Claim 4. *If* $\{p\}\in C(A_1,\ldots, A_n)$ *and* $\{\sim p\}\notin C(A_1,\ldots, A_n)$, then $|N_{\{p\}}|\geq\alpha$ (and $|N_{\{\sim p\}}|<\alpha$.)
Proof of Claim 4. By Claims 2 and 3, if $|N_{\{\sim p\}}|\geq\alpha$, then $\{\sim p\}\in C(A_1,\ldots, A_n)$ and $\{p\}\notin C(A_1,\ldots, A_n)$. Since $\{p\}\in C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$, then $|N_{\{\sim p\}}|<\alpha$. Suppose that $|N_{\{p\}}|<\alpha$. Then by Claim 1, $\{\sim p\}\notin C(A_1,\ldots, A_n)$ and $\{p\}\notin C(A_1,\ldots, A_n)$. Hence, since $\{p\}\in C(A_1,\ldots, A_n)$ and $\{\sim p\}\notin C(A_1,\ldots, A_n)$, then $|N_{\{p\}}|\geq\alpha$. QED.

Claim 5. For any profile $(A_1,\ldots, A_n)$ and any proposition p, $\{p\}\in C(A_1,\ldots, A_n)$ iff $|N_{\{p\}}|\geq\alpha$.
Proof of Claim 5. By proposition-wise weak consistency and Claim 4, if $\{p\}\in C(A_1,.., A_n)$, then $|N_{\{p\}}|\geq\alpha$. On the other hand, by Claim 3, if $|N_{\{p\}}|\geq\alpha$ then $\{p\}\in C(A_1,\ldots, A_n)$. QED.

Proof of Theorem 4.2.
As in former cases, let us focus on the 'if, then' part. In addition, given a profile $(A_1,..,A_n)$ and a pair of judgment sets $A, A^*\in Q^A$, let $(A'_1,.., A'_n)$ be any profile such that $N_A=N'_{A^*}$, $N'_A=N_{A^*}$, and for the remaining judgment sets A' in Q^A, $N_{A'}=N'_{A'}$.

Claim 1. *If* $|N_A|=|N_{A^*}|$ *and* $A^*\in C(A_1,.., A_n)$, *then* $A\in C(A_1,.., A_n)$.
Proof of Claim 1. Assume to the contrary that $|N_A|=|N_{A^*}|$, $A^*\in C(A_1,.., A_n)$ and $A\notin C(A_1,.., A_n)$. Notice that in this case $(A'_1,.., A'_n)$ is a permutation of $(A_1,.., A_n)$. By anonymity, $A^*\in C(A'_1,.., A'_n)$ and $A\notin C(A'_1,.., A'_n)$. But, in contradiction, contextual systematicity implies that $A^*\notin C(A'_1,.., A'_n)$ and $A\in C(A'_1,.., A'_n)$. Hence, if $|N_A|=|N_{A^*}|$ and $A^*\in C(A_1,.., A_n)$, then $A\in C(A_1,.., A_n)$. QED.
Claim 2. *If* $|N_A|>|N_{A^*}|$ *and* $A^*\in C(A_1,.., A_n)$, *then* $A\in C(A_1,.., A_n)$.

Proof of Claim 2. Assume now that $|N_A|>|N_{A*}|$, $A^*\in C(A_1,.., A_n)$ and $A\notin C(A_1,.., A_n)$. By contextual systematicity, $A\in C(A'_1,.., A'_n)$ and $A^*\notin C(A'_1,.., A'_n)$. Let $(A''_1,.., A''_n)$ be another profile such that (1) $|N''_{A*}|=|N'_{A*}|=|N_A|$ and $|N''_A|=|N'_A|=|N_{A*}|$, (2) $N_{A*}\subset N''_{A*}$; (3) $N''_A\subset N_A$; (4) $N''_{A*}\cup N''_A=N_{A*}\cup N_A$; and (5) for the remaining judgment sets $A''\in Q^A$, $N_{A''}=N''_{A''}$. Notice that $(A''_1,.., A''_n)$ is a permutation f of $(A'_1,.., A'_n)$ such that for any individual i, $A'_i=A''_{f(i)}$. Since $A\in C(A'_1,.., A'_n)$ and $A^*\notin C(A'_1,.., A'_n)$, anonymity implies that $A\in C(A''_1,.., A''_n)$ and $A^*\notin C(A''_1,.., A''_n)$. But then, since 1) $N''_A\subset N_A$ and, in addition, 2) $|N''_{A*}|>|N_{A*}|$ and $N''_{A''}=N_{A''}$ for any judgment set in Q^A such that $A''\neq A$ and $A''\neq A^*$, then set-wise weak plurality positive responsiveness implies that $A\in C(A_1,.., A_n)$, in contradiction to the above assumption that $A^*\in C(A_1,.., A_n)$ and $A\notin C(A_1,.., A_n)$. QED.

Claim 3. *If* $|N_A|>|N_{A*}|$, *then* $A^*\notin C(A_1,.., A_n)$.
Proof of Claim 3. Let us suppose that $|N_A|>|N_{A*}|$ and $A^*\in C(A_1,.., A_n)$. By Claim 2, $A\in C(A'_1,.., A'_n)$. However, since $A^*\in C(A_1,.., A_n)$, the set-wise weak plurality positive responsiveness implies then that $A\notin C(A'_1,.., A'_n)$. Contradiction. Therefore, if $|N_A|>|N_{A*}|$, then $A^*\notin C(A_1,.., A_n)$. QED.

Claim 4. *If* $|N_A|\geq|N_{A'}|$ *for all* $A'\in Q^A$, *then* $A\in C(A_1,.., A_n)$.
Proof of Claim 4. Assume that $|N_A|\geq|N_{A'}|$ for all $A'\in Q^A$. Since C is contextually complete, there is some $A''\in Q^A\cap C(A_1,.., A_n)$. But then Claims 1 and 2 imply that $A\in C(A_1,.., A_n)$. Hence, if $|N_A|\geq|N_{A'}|$ for all $A'\in Q^A$, then $A\in C(A_1,.., A_n)$. Suppose now that $A\in C(A_1,.., A_n)$ and, by contradiction, suppose that $|N_{A'}|>|N_A|$ for some $A'\in Q^A$. Claim 3 implies that $A\notin C(A_1,.., A_n)$. Contradiction. Then, if $A\in C(A_1,.., A_n)$, then $|N_A|\geq|N_{A'}|$ for all $A'\in Q^A$. QED.

Proof of Theorem 4.3. For the same reason as with the former characterization theorems, we need to prove only the 'if, then' part. Given a profile $(A_1,.., A_n)$ and a pair of judgment sets $A, A^*\in Q^A$, let $(A'_1,.., A'_n)$ be any profile such that $N_A=N'_{A*}$, $N'_A=N_{A*}$, and for the remaining judgment sets A' in Q^A, $N_{A'}=N'_{A'}$.

Claim 1. *If* $|N_A|=|N_{A*}|$ *and* $A^*\in C(A_1,.., A_n)$, *then* $A\in C(A_1,.., A_n)$.
Proof of Claim 1. (See the proof of Claim 1 in the proof of Theorem 4.2).

Claim 2. *If* $|N_A|>|N_{A*}|$ *and* $A^*\in C(A_1,.., A_n)$, *then* $A\in C(A_1,.., A_n)$.
Claim 3. *If* $|N_A|>|N_{A*}|$, *then* $A^*\notin C(A_1,.., A_n)$.
Proof of Claims 2 and 3. (Like the proofs of Claims 2 and 3 in the proof of Theorem 4.2, substituting set-wise plurality positive responsiveness for set-wise weak plurality positive responsiveness).

Claim 4. *If* $|N_A|>|N_{A'}|$ *for all* $A'\in Q^A$, *then* $A\in C(A_1,.., A_n)$.
Proof of Claim 1. Assume that $|N_A|>|N_{A'}|$ for all $A'\in Q^A$ and $A\notin C(A_1,.., A_n)$. Since C is contextually semi-complete, there is an $A''\in Q^A$ such that $A''\in C(A_1,.., A_n)$. If $A''\neq A$, then $|N_A|>|N_{A''}|$, so $A''\notin C(A_1,.., A_n)$ by Claim 3. Contradiction. QED.

Claim 5. If $A\in C(A_1,.., A_n)$ *then* $|N_A|>|N_{A'}|$ *for all* $A'\in Q^A$.

Proof of Claim 2. Suppose $A \in C(A_1,.., A_n)$, and by contradiction suppose $|N_{A'}| \geq |N_A|$ for some $A' \in Q^A$. Then Claims 1 and 2 say $A' \in C(A_1,.., A_n)$ also. But this contradicts union-wise weak consistency. QED.